Theory and algorithms for shape reconstruction from apparent contours

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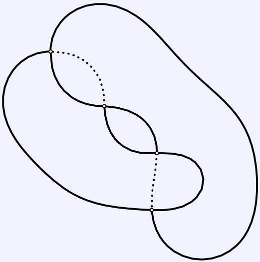
joint book with V. Beorchia (Univ. Trieste, Italy), M. Paolini, F. Pasquarelli (Univ. Cattolica Brescia, Italy)
[**BBPP**]: *Shape Reconstruction from Apparent Contours. Theory and Algorithms*, Computational Imaging and Vision, Springer-Verlag, to appear.

Motivations

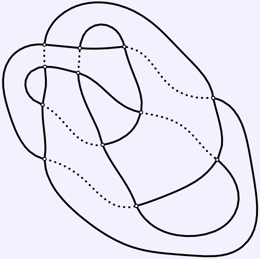
Variational model of
M. Nitzberg, D.Mumford: The 2.1-D sketch, 1990
M. Nitzberg, D. Mumford, T. Shiota: Lect. Not. Comp. Sci. 662, Springer-Verlag, 1993
reconstruct a given grey level, and its hidden parts, minimizing an action defined on plane curves, penalizing the length and the curvature of the contours, and depending on a notion of *ordering* between the objects in the scene. A minimal configuration carries a "depth" order, saying which object in the 3D shape is in front of the other, and which is back.

Restriction of the model related to occlusions: it enforces a "global" ordering on the objects, considered as "flat silhouettes" at constant distance from the observer.
This *excludes* situations like:

two objects of the 3D shape overlap in opposite order in different locations



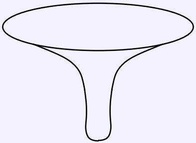
a single object self-overlaps



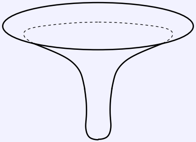
There is an action functional defined on plane curves, which can take into account the overlapping regions and self-occlusions. This action is *defined on apparent contours*; this motivates our study of apparent contours, since they enter in the domain of the functional. Minimization of this functional can also gives a possible way to reconstruct the hidden contours. See [**BBP**], G. Bellettini, V. Beorchia, M. Paolini: J. Math. Imaging Vision **32** (2008), 265–291

The action penalizes the length and the curvature of the contours, and the total number of nodes. It tends to minimize the invisible part of the contours.

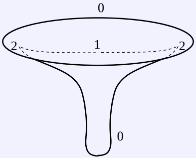
jump of the initial grey-level



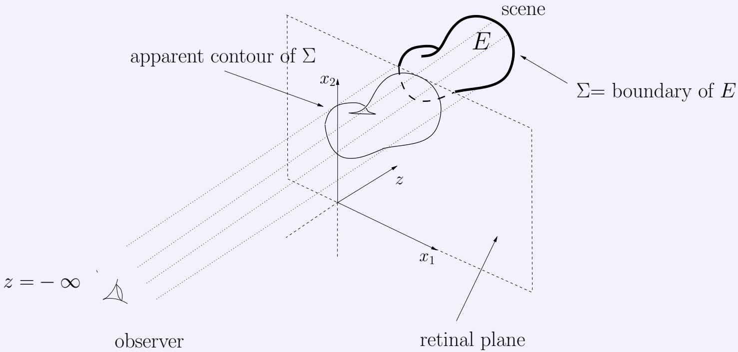
completion given by Nitzberg-Mumford: the 3D shape consists of two objects one in front of the other



another possible completion given by the new action: the 3D shape is a mushroom

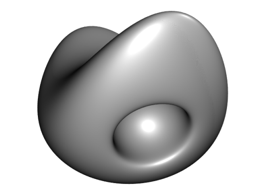


Apparent contours, labelling and visible contours

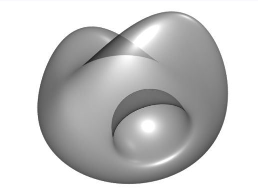


See for instance the book **J.J Koenderink**: Solid Shape, MIT Press, Cambridge 1990

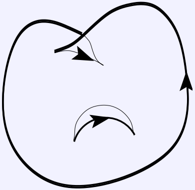
3D shape $E \subset \mathbb{R}^3$, Σ its boundary. E is not necessarily connected, but it is *smooth*



3D shape considered semi-transparent



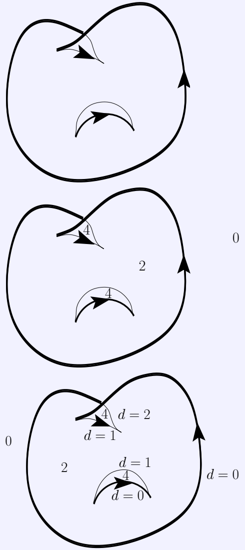
apparent contour $\mathrm{appcon}(\Sigma)$



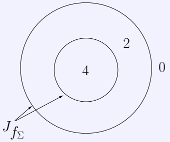
apparent contour

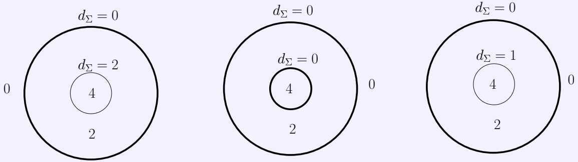
total number of intersections with the light ray, on the regions. It is denoted by $f = f_\Sigma \in 2\mathbb{N}$; deduced from the orientation, as twice the (total) winding number

labelled apparent contour: the labelling is $d = d_\Sigma \in \mathbb{N}$, defined on the arcs. $\{d = 0\}$ is the **visible** part



f alone cannot identify a three-dimensional scene: consider

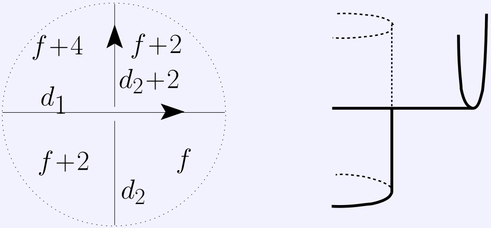




left: large sphere in front of a small one
center: large sphere behind the small one
right: large sphere with a hole inside

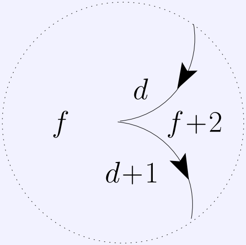
compatibility of the labelling around a crossing

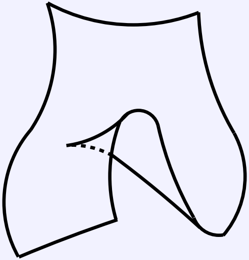
$0 \leq d_1 \leq d_2 \leq f$



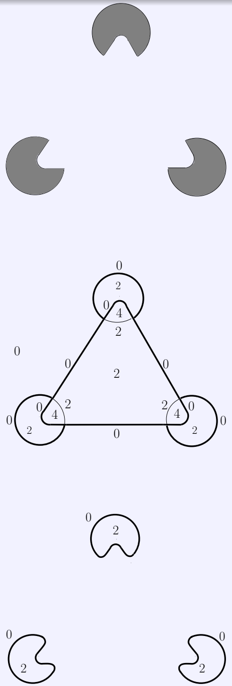
compatibility of the labelling around a cusp

$0 \leq d < f$





See [**BBP**] for more.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Warning: a suitable notion of *stability* is required. This is a delicate and important point which we do not want to discuss here. Such a stability will be assumed from now on.

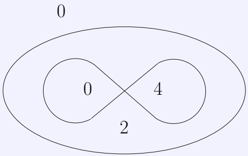
Singularities (vertices, also called nodes) of the graphs:

- apparent contour: crossings and cusps
- visible apparent contour: T-junctions and terminal points

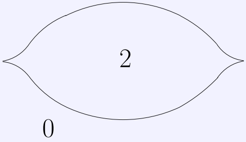
Questions:

- is a plane graph with crossings and cusps the apparent contour of a 3D shape?
- when an oriented plane graph with only T-junctions and terminal points is the visible part of a labelled apparent contour?
- can we recognize, looking at the apparent contours, when two 3D shapes are equivalent (ambient isotopic)?
- in the class of equivalent 3D shapes, can we find one having the "simplest" apparent contour?
- can we authomatize these issues in a computer program?

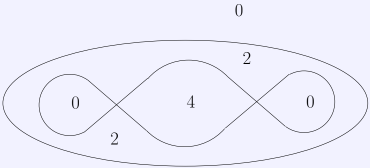
graph which is **not** apparent contour of a 3D shape. Remark: there is no way to put a consistent labelling *d*



graph which is **not** apparent contour of a 3D shape. Also here, no way to put a consistent labelling



graph which **is** apparent contour of a 3D shape



Theorem (Existence, [BBP])

Given an oriented plane graph G with cusps and crossings, endowed with a labelling $d : G \to \mathbb{N}$ satisfying all compatibility conditions, there exists a smooth 3D shape E such that

$$G = \mathrm{appcon}(\Sigma), \qquad d = d_{\Sigma}$$

where Σ denotes the boundary of E.

Related references:

L.R. Williams: Ph.D. dissertation, Dept. of Computer Science, Univ. of Massachusetts, Amherst, Mass. 1994

L.R. Williams: Int. J. Computer Vision **23** (1997), 93–108

Theorem (Uniqueness, [BBP])

Σ is unique, up to transformations of $\mathbb{R}^3$ which do not change the order and the number of intersections of the manifold with the light rays emanating from the projection plane (and therefore do not modify the corresponding labelled apparent contour).

- The proof, based on a cut and paste technique, furnishes an embedded smooth manifold Σ, but not the "roundest" way to embed it in the ambient space $\mathbb{R}^3$. This probably would require a variational argument on surfaces, beyond our scopes. See for instance **O.A. Karpenko, J.F. Hughes**, SIGGRAPH 2006, 589-598, New York, for "round" embeddings.
- The reconstruction problem is completely solved from an algorithmic point of view, using the program `appcontour` developed in [**BBPP**].
- realize how the 3D shape "looks like" can be difficult.

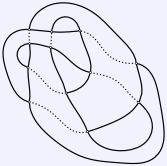
`appcontour` in [**BBPP**] reconstructs the topological structure of $\Sigma = \partial E$, in particular the number of connected components of Σ and the Euler-Poincaré characteristic of each of them, together with information allowing to distinguish, for example, between a hollow sphere and two mutually external spheres.

Theorem (Characteristic from the apparent contour, [BBP])

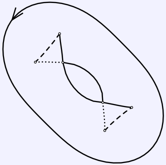
Let (G, d) be a labelled graph and $\Sigma = \partial E$ be the reconstructed 3D shape. Then the total Euler-Poincaré characteristic $\chi(\Sigma)$ of Σ can be computed solely from the apparent contour.

In the special case where ∂E is connected, we deduce the Euler-Poincaré characteristic of the solid set E and of its complement $\mathbb{R}^3 \setminus E$ from the apparent contour G. These computations are implemented in `appcontour`.

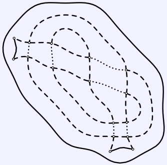
E_0 knotted solid torus



E_1 standard solid torus



E_2 be a sphere with a knotted gallery connecting two removed disks (knotted anti-torus)



$$\chi(\partial E_0) = \chi(\partial E_1) = \chi(\partial E_2)$$
$$\chi(E_0) = \chi(E_1) = \chi(E_2)$$
$$\chi(\mathbb{R}^3 \setminus E_0) = \chi(\mathbb{R}^3 \setminus E_1) = \chi(\mathbb{R}^3 \setminus E_2)$$

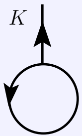
But they are not ambient-isotopic one each other.

Other invariants (of the apparent contour G, and of the 3D shape E) can be considered; some of them are implemented in `appcontour`. Most notably, the first fundamental group of $\mathbb{R}^3 \setminus E$. In order to recognize the shape, it is important to simplify its apparent contour; this can be done using a suitable set of elementary *moves*: this is maybe the main feature of `appcontour`.

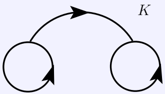
The topological structure of an apparent contour is invariant under smooth deformations of the plane. The software code is devised in such a way to be insensitive to the particular embedding of the apparent contour in the plane.

Completion of visible contours

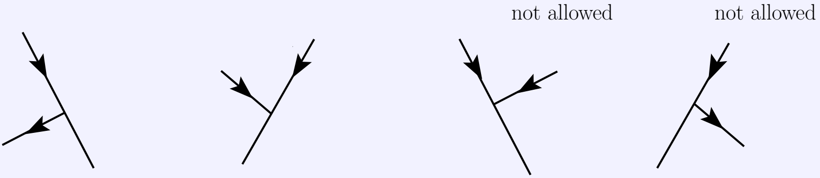
terminal point adjacent to the exterior region: K **cannot** be visible part of an apparent contour



external region on the left of an arc: K **cannot** be visible part of an apparent contour



local orientations allowed and **not** allowed at a T-junction for being visible part of an apparent contour



See [**BBP1**], G. Bellettini, V. Beorchia, M. Paolini: SIAM J. Imaging Sci. **2** (2009), 777-799.

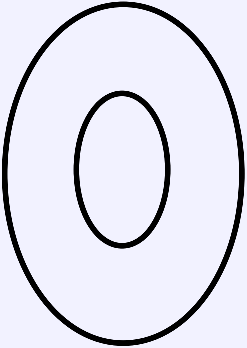
Theorem (Completion, [BBP1])

Let K be an oriented plane graph with T-junctions and terminal points. Suppose we do not fall in the examples of the previous slides. Then there exists a labelled apparent contour (G, d) such that

$$K = \{d = 0\}.$$

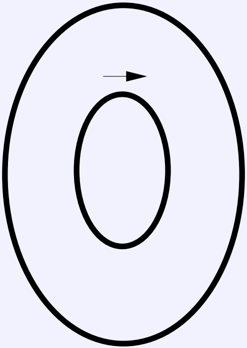
- The proof is constructive, and implemented in `visiblecontour`, part of `appcontour` in [**BBPP**]. It is based on a Morse description of the various graphs.
- The aim of the completion theorem is not to provide the "simplest" completion of K, whatever simplest could mean. The scope of the result is to show that the hypotheses are sharp, and to allow us to construct at least one completion.

Annulus I



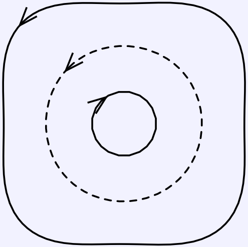
The visible contour, deliberately unoriented. The internal circle cannot be implicitly oriented, since both orientations lead to an admissible visible contour. This leads `visiblecontour` to the error message `Insufficient orienting information`

Annulus II



clockwise orientation of the internal circle. We impose $f = 0$ only in the exterior region. The resulting reconstruction does not (and cannot) correspond to a torus

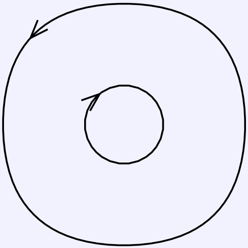
The result given by `visiblecontour` in [**BBPP**] is:



? Answer: a visible torus, with an intermediate sphere behind.

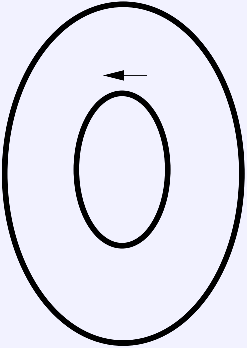
Annulus III

We force $f = 0$ in the smaller region. `visiblecontour` trivially reconstructs the apparent contour of a torus with no hidden arcs



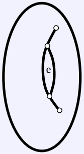
Annulus IV

The internal circle is now oriented counterclockwise, hence the internal region cannot to be part of the background. `visiblecontour` reconstructs the apparent contour with no hidden arcs, corresponding a small sphere in front of a bigger one

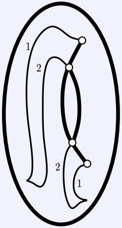


See [**BBPP**] for more.

Torus I



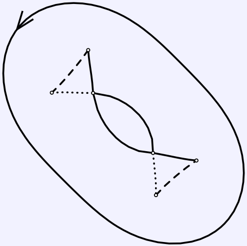
we force the small internal e-region to be part of the background. `visiblecontour` produces



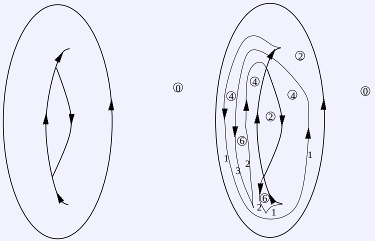
? Answer: apparent contour of a torus.

Why? The resulting Morse description can be read by `appcontour` using the `unix` command line
visible example.morse — contour printmorse — showcontour

the output [**BBPP**] is



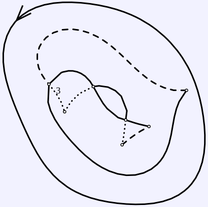
We remove the marking of the small internal region as part of the background.



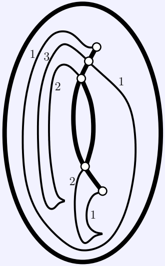
(a) (b)

The right picture shows the result of the constructive proof of the completion theorem given in [**BBP1**].

`visiblecontour` produces (equivalently)



or also



? A deformed 3D sphere: this can be seen, for instance, using `appcontour`, by means of the elementary *moves*.

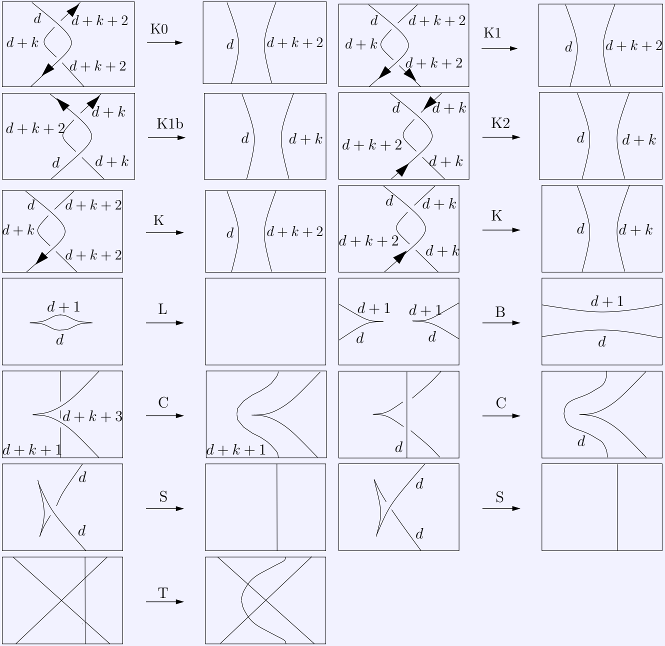
Reidemeister-type moves on apparent contours

The above examples have shown that it may be difficult to recognize the 3D shape looking at its labelled apparent contour. It is therefore necessary to find moves that simplify the apparent contours, remaining inside the class of equivalence of the 3D shape. The same problem is well-known in the (simpler) setting of knot theory, and the moves are called Reidemeister moves.

Theorem (Completeness)

Two 3D shapes are ambient isotopic if and only if their apparent contours can be connected using only a finite set of elementary moves (or Reidemeister-type moves) on labelled apparent contours and a finite number of smooth planar isotopies.

Reference: [**BBP2**], G. Bellettini, V. Beorchia, M. Paolini: Atti Accad. Naz. Lincei Cl. Sci. Fis. Mat. Natur. Rend. Lincei (9) Mat. Appl. **22** (2011), 1–19



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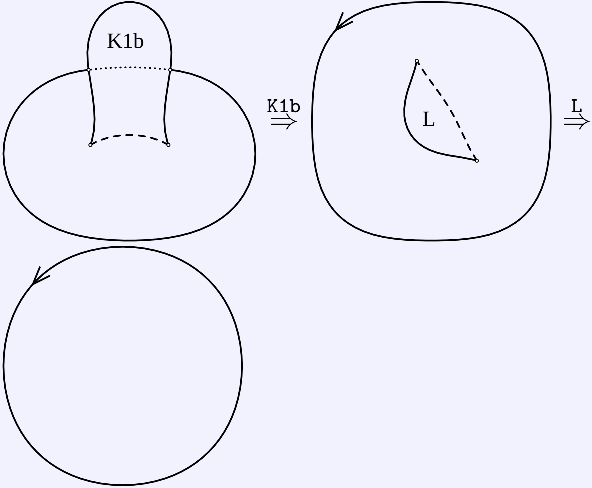
- The proof is based on the classification of singularities of stable maps, due to important works of **Whitney**, **Thom**, **Arnold** and their schools.
- The main ability of `appcontour` is to manipulate these moves and their inverses on apparent contour, in order to recognize a 3D shape. For instance, action `rules` requests a list of the simplifying Reidemeister-type moves or composite moves that can be legally applied to the given contour.

Example from [**BBPP**]: simplifying the mushroom



Elimination of cusps

Theorem (Elimination, [BBPP])

Up to $\mathbb{R}^3$-ambient isotopies, any smooth closed surface embedded in $\mathbb{R}^3$ has an apparent contour without cusps.

The proof is based on the application of various combinations of some of the elementary moves and their inverses on labelled apparent contours.
The elimination of all cusps is obtained, in some cases, at the expenses of increasing the number of crossings.

Conclusions

- complete variational study of the action functional on apparent contours; genericity is lost in the (weak) limit of generic apparent contours
- extend the theory to the case of polyhedral (Lipschitz) 3D scenes (Y-junctions)
- variational study of an action making the reconstructed shape as "round" as possible
- deepen the study of apparent contours and the moves in case of immersed manifold, or even in more generality. The literature of differential topology and singularity theory is very rich. See for instance the work of J.S. Carter, S. Kamada, M. Saito. `appcontour` is already written so to *partially* consider these cases
- understand better the invariants of 3D shapes, in the spirit of what has been done so far for (tubular neighbourhoods of) knots