Models in computer vision often require to estimate continuous transformations of the 3D space, i.e. elements of a Lie group. This demands a consistent framework for statistics on Lie groups. It is known that there is no fully consistent (bi-invariant) Riemannian metric on most Lie groups [1]. We investigate here the existence of bi-invariant pseudo-Riemannian metrics and we present an algorithm to compute such a pseudo-metric on a Lie group if it exists. Unfortunately results show that most Lie groups do not possess a bi-invariant pseudo-metric in general, although the class of such groups is larger than for the Riemannian case.

**Consistent statistical framework on Lie groups: Example of the mean**

**Question 1:** which Lie groups do we add by asking for a bi-invariant pseudo-metric instead of a bi-invariant metric?
**Question 2:** is the pseudo-Riemannian setting rich enough to provide a consistent framework for statistics on Lie groups?

**Algorithm to compute bi-invariant pseudo-metrics on Lie groups**

**Theorem [2]: Structure of a Lie algebra that admits a bi-invariant pseudo-metric**

\( g \) has a bi-invariant pseudo-metric iff its adjoint representation decomposition \( g = B^0 \oplus B^3 \) where:
- \( B^0 = \{ b \in g | ad(b) = \lambda b \} \) is a vector space
- \( \forall \lambda, [g, B^3] \subset B^3 \), i.e. \( B^3 \) an ideal

**Structure of Lie groups with a bi-invariant pseudo-metric**

**Algorithms to compute bi-invariant pseudo-metric on \( g \) in case of existence [4]**

1. \( B_i^3 = \text{1-dim} \) ?
2. \( B_i^3 \text{ simple?} \)
3. \( B_i^8 = S \oplus S^* \) ?
4. \( B_i^8 = \mathbb{W} \oplus S \oplus S^* \) ?

Else: EXIT, thus no bi-invariant \(<,\>.

**Algorithm to compute bi-invariant pseudo-metric on \( g \) in case of existence [4]**

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And recombination on \( g \):

\[ b_1 \oplus \ldots \oplus b_N, b_1^* \oplus \ldots \oplus b_N^* \]

**Comparison**

**Structure of Lie groups with a bi-invariant pseudo-metric**

**Results**

1. \( B_i^3 = \text{1-dim}? \)
2. \( B_i^3 \text{ simple?} \)
3. \( B_i^8 = S \oplus S^*? \)
4. \( B_i^8 = \mathbb{W} \oplus S \oplus S^*? \)

EXIT: No

References:

Answer 2: The class of Lie groups with bi-invariant pseudo-metric is not large enough to try to using the pseudo-Riemannian setting [4].